Homework Problems: Concepts of Steady State and Stability

[1] Consider the difference equation: $y_{t+1} = \alpha y_t$.
(a) Write down the expressions for $F(t, y_{t+1}, y_t; \alpha)$ and $f(t, y_t; \alpha)$.
(b) What kind of difference equation is it? Linear or non-linear? What is the order?
(c) Is it autonomous? Calculate $\Delta y$, and show whether or not it depends on $t$.
(d) Suppose we have the initial condition $y_0 = y^0$. Solve the equation for $g(t, y^0; \alpha)$, iteratively starting at $t = 0$.
(e) Graph time-paths for $\alpha = 2$ and $y^0 = 0.01$. What if $y^0 = 0.02$?
(f) What happens if $\alpha = 0.5$. Draw time-paths for both $y^0 = 0.01$ and $y^0 = 0.02$?

[2] Consider the equation, $m y_{t+1} - n y_t = 0$.
(a) Show if it is or is not homogeneous?
(b) What is the solution function $g(t, y^0; m, n)$ of this equation, when $y_0 = y^0$?
(c) Draw a phase diagram and a time path for each of the following cases (pictures don’t have to be exactly to the scale):
   (i) $n = 0.25$, $m = 0.5$
   (ii) $n = -1.0$, $m = 2.0$
   (iii) $n = 4.0$, $m = 2.0$
   (iv) $n = 5$, $m = -2.5$
   (v) $n = 2.0$, $m = 2.0$
   (vi) $n = 1.0$, $m = -1.0$
(d) Write down the $\delta-\epsilon$ definition of asymptotic stability. Discuss each of the above cases in question (c) in light of this definition.