**CRRA Utility Function in OLG Model**

Consider the Diamond OLG model where the utility function of the representative two-period-lived household is,

$$U(c_t, c_{t+1}) = \frac{c_t^{1-\sigma}}{1-\sigma} + \beta \frac{c_{t+1}^{1-\sigma}}{1-\sigma}, \quad \sigma > 0, \quad \sigma \neq 1, \quad \beta \equiv 1/(1+\theta).$$

The firms are competitive with Cobb-Douglas production functions,

$$f(k_t) = k_t^\alpha, \quad \alpha \in (0,1).$$

The population grows at a constant rate $n$.

(i) Set up the household’s problem. Calculate their optimal consumptions and saving.

(ii) Calculate $w'(k_t)$ and $r'(k_t)$. Can you determine the signs of $w'(k_t)$ and $r'(k_t)$?

(iii) Show that, whether $\frac{dk_{t+1}}{dk_t}$ is $>0$ or $<0$ depends of the elasticity of substitution between the two periods consumptions.

---

**CRRA Utility Function in OLG Model: The Central Planner’s Version**

For this problem, you can confine your analysis to steady state only. Consider the Diamond OLG model where the utility function of the representative two-period-lived household is,

$$U(c_t, c_{t+1}) = \frac{c_t^{1-\sigma}}{1-\sigma} + \beta \frac{c_{t+1}^{1-\sigma}}{1-\sigma}, \quad \sigma > 0, \quad \sigma \neq 1, \quad \beta \equiv 1/(1+\theta).$$

The firms are competitive with Cobb-Douglas production functions,

$$f(k_t) = k_t^\alpha, \quad \alpha \in (0,1).$$

The population grows at a constant rate $n$.

(i) Set up the central planner’s problem. Derive the first order conditions.

(ii) Calculate $k^*_g$, the golden rule level of capital stock.

---

**OLG Model with Depreciation**

Consider the Diamond OLG model where the utility function of the representative two-period-lived household is separable on the two periods consumptions $(c_{it}, c_{2t+1})$ with the discount factor $\beta \equiv 1/(1+\theta)$. The population grows at a constant rate $n$. The firms are competitive with constant returns to scale production functions. The depreciation rate of capital is $\delta$, so that $r = f'(k_t) - \delta$.

(i) Derive the Euler equation for the representative household. How does it differ from the model where there is no depreciation of capital (i.e. the model that we have been working on in class)?

(ii) Again, compared to the model without depreciation, does adding depreciation alter the relationship between $k_{t+1}$ and $k_t$?

(iii) Suppose we can write $s_t(w_t, r_{t+1}) = mpc(r_{t+1})w_t$ for each $\theta$. Also, suppose $\frac{\partial s_t}{\partial r_{t+1}} > 0$. Show that saving loci is lower in the economy with depreciation compared to the economy with no depreciation.