Bonds & Stocks
Definition of a Bond

• A bond is a legally binding agreement between a borrower and a lender that specifies the:
  – Par (face) value
  – Coupon rate
  – Coupon payment
  – Maturity Date

• The yield to maturity is the required market interest rate on the bond.
How to Value Bonds

• Primary Principle:
  – Value of financial securities = PV of expected future cash flows

• Bond value is, therefore, determined by the present value of the coupon payments and par value.

• Interest rates are inversely related to present (i.e., bond) values.
The Bond Pricing Equation

\[
\text{Bond Value} = C \left[ 1 - \frac{1}{(1+R)^T} \right] + \frac{FV}{(1+R)^T}
\]
Pure Discount Bonds

- Make no periodic interest payments (coupon rate = 0%)
- The entire yield to maturity comes from the difference between the purchase price and the par value.
- Cannot sell for more than par value
- Sometimes called zeroes, deep discount bonds, or original issue discount bonds (OIDs)
- Treasury Bills and principal-only Treasury strips are good examples of zeroes.
Pure Discount Bonds

Information needed for valuing pure discount bonds:
- Time to maturity ($T$) = Maturity date - today’s date
- Face value ($F$)
- Discount rate ($r$)

Present value of a pure discount bond at time 0:

$$PV = \frac{FV}{(1 + R)^T}$$
Pure Discount Bond: Example

Find the value of a 30-year zero-coupon bond with a $1,000 par value and a YTM of 6%.

\[
PV = \frac{FV}{(1 + R)^T} = \frac{$1,000}{(1.06)^{30}} = $174.11
\]
Level Coupon Bonds

- Make periodic coupon payments in addition to the maturity value
- The payments are equal each period. Therefore, the bond is just a combination of an annuity and a terminal (maturity) value.
- Coupon payments are typically semiannual.
- Effective annual rate (EAR) = \((1 + \frac{R}{m})^m - 1\)
Level Coupon Bond: Example

- Consider a U.S. government bond with a 6 3/8% coupon that expires in December 2010.
  - The *Par Value* of the bond is $1,000.
  - *Coupon payments* are made semi-annually (June 30 and December 31 for this particular bond).
  - Since the *coupon rate* is 6 3/8%, the payment is $31.875.
  - On January 1, 2006 the size and timing of cash flows are:

<table>
<thead>
<tr>
<th>Date</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/1/06</td>
<td>$31.875</td>
</tr>
<tr>
<td>6/30/06</td>
<td>$31.875</td>
</tr>
<tr>
<td>12/31/06</td>
<td>$31.875</td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
<tr>
<td>6/30/10</td>
<td>$31.875</td>
</tr>
<tr>
<td>12/31/10</td>
<td>$1,031.875</td>
</tr>
</tbody>
</table>
Level Coupon Bond: Example

• On January 1, 2010, the required annual yield is 5%.

\[ PV = \frac{31.875}{.05/2} \left[ 1 - \frac{1}{(1.025)^{10}} \right] + \frac{1,000}{(1.025)^{10}} = 1,060.17 \]
Bond Example: Calculator

Find the present value (as of January 1, 2006), of a 6 3/8% coupon bond with semi-annual payments, and a maturity date of December 2010 if the YTM is 5%.

\[
PMT = \frac{1,000 \times 0.06375}{2} = 31.875
\]

\[
PV = -1,060.17
\]

\[
N = 10
\]

\[
I/Y = 2.5
\]

\[
FV = 1,000
\]
Bond Pricing with a Spreadsheet

- There are specific formulas for finding bond prices and yields on a spreadsheet.
  - \text{PRICE}(\text{Settlement}, \text{Maturity}, \text{Rate}, \text{Yld}, \text{Redemption}, \text{Frequency}, \text{Basis})
  - \text{YIELD}(\text{Settlement}, \text{Maturity}, \text{Rate}, \text{Pr}, \text{Redemption}, \text{Frequency}, \text{Basis})
  - Settlement and maturity need to be actual dates
  - The redemption and Pr need to be given as % of par value

- Click on the Excel icon for an example.
Consols

• Not all bonds have a final maturity.
• British consols pay a set amount (i.e., coupon) every period forever.
• These are examples of a *perpetuity*.

\[
P V = \frac{C}{R}
\]
Bond Laws

• Bond **prices** and market **interest rates** move in **opposite directions**.
• When coupon rate = YTM, price = par value
• When coupon rate > YTM, price > par value (premium bond)
• When coupon rate < YTM, price < par value (discount bond)
YTM and Bond Value

When the YTM < coupon, the bond trades at a premium.

When the YTM = coupon, the bond trades at par.

When the YTM > coupon, the bond trades at a discount.
Bond Example Revisited

- Using our previous example, now assume that the required yield is 11%.
- How does this change the bond’s price?

\[
PV = \frac{$31.875}{.11/2} \left[ 1 - \frac{1}{(1.055)^{10}} \right] + \frac{$1,000}{(1.055)^{10}} = $825.69
\]
Computing Yield to Maturity

- Yield to maturity is the rate implied by the current bond price.
- Finding the YTM requires trial and error if you do not have a financial calculator and is similar to the process for finding $R$ with an annuity.
- If you have a financial calculator, enter $N$, $PV$, $PMT$, and $FV$, remembering the sign convention ($PMT$ and $FV$ need to have the same sign, $PV$ the opposite sign).
YTM with Annual Coupons

- Consider a bond with a 10% annual coupon rate, 15 years to maturity, and a par value of $1,000. The current price is $928.09.
  - Will the yield be more or less than 10%?
  - N = 15; PV = -928.09; FV = 1,000; PMT = 100
  - CPT I/Y = 11%
YTM with Semiannual Coupons

- Suppose a bond with a 10% coupon rate and semiannual coupons has a face value of $1,000, 20 years to maturity, and is selling for $1,197.93.
  - Is the YTM more or less than 10%?
  - What is the semiannual coupon payment?
  - How many periods are there?
  - \( N = 40; PV = -1,197.93; PMT = 50; FV = 1,000; \) CPT I/Y = 4% (Is this the YTM?)
  - YTM = 4%*2 = 8%
Bond Market Reporting

- Primarily over-the-counter transactions with dealers connected electronically
- Extremely large number of bond issues, but generally low daily volume in single issues
- Makes getting up-to-date prices difficult, particularly on a small company or municipal issues
- Treasury securities are an exception
Treasury Quotations

- What is the coupon rate on the bond?
- When does the bond mature?
- What is the bid price? What does this mean?
- What is the ask price? What does this mean?
- How much did the price change from the previous day?
- What is the yield based on the ask price?
The Present Value of Common Stocks

• The value of any asset is the present value of its expected future cash flows.

• Stock ownership produces cash flows from:
  – Dividends
  – Capital Gains

• Valuation of Different Types of Stocks
  – Zero Growth
  – Constant Growth
  – Differential Growth
Case 1: Zero Growth

- Assume that dividends will remain at the same level forever

\[ \text{Div}_1 = \text{Div}_2 = \text{Div}_3 = \cdots \]

- Since future cash flows are constant, the value of a zero growth stock is the present value of a perpetuity:

\[ P_0 = \frac{\text{Div}_1}{(1 + R)^1} + \frac{\text{Div}_2}{(1 + R)^2} + \frac{\text{Div}_3}{(1 + R)^3} + \cdots \]

\[ P_0 = \frac{\text{Div}}{R} \]
Case 2: Constant Growth

Assume that dividends will grow at a constant rate, \( g \), forever, i.e.,

\[
\begin{align*}
\text{Div}_1 &= \text{Div}_0 (1 + g) \\
\text{Div}_2 &= \text{Div}_1 (1 + g) = \text{Div}_0 (1 + g)^2 \\
\text{Div}_3 &= \text{Div}_2 (1 + g) = \text{Div}_0 (1 + g)^3 \\
&\quad \vdots
\end{align*}
\]

Since future cash flows grow at a constant rate forever, the value of a constant growth stock is the present value of a growing perpetuity:

\[
P_0 = \frac{\text{Div}_1}{R - g}
\]
Constant Growth Example

• Suppose Big D, Inc., just paid a dividend of $.50. It is expected to increase its dividend by 2% per year. If the market requires a return of 15% on assets of this risk level, how much should the stock be selling for?

\[ P_0 = \frac{.50(1+.02)}{(.15 - .02)} = $3.92 \]
Case 3: Differential Growth

• Assume that dividends will grow at different rates in the foreseeable future and then will grow at a constant rate thereafter.

• To value a Differential Growth Stock, we need to:
  – Estimate future dividends in the foreseeable future.
  – Estimate the future stock price when the stock becomes a Constant Growth Stock (case 2).
  – Compute the total present value of the estimated future dividends and future stock price at the appropriate discount rate.
Case 3: Differential Growth

Assume that dividends will grow at rate $g_1$ for $N$ years and grow at rate $g_2$ thereafter.

\[
\begin{align*}
\text{Div}_1 &= \text{Div}_0 (1 + g_1) \\
\text{Div}_2 &= \text{Div}_1 (1 + g_1) = \text{Div}_0 (1 + g_1)^2 \\
& \quad \vdots \\
\text{Div}_N &= \text{Div}_{N-1} (1 + g_1) = \text{Div}_0 (1 + g_1)^N \\
\text{Div}_{N+1} &= \text{Div}_N (1 + g_2) = \text{Div}_0 (1 + g_1)^N (1 + g_2) \\
& \quad \vdots
\end{align*}
\]
Case 3: Differential Growth

Dividends will grow at rate $g_1$ for $N$ years and grow at rate $g_2$ thereafter.

\[
\begin{align*}
\text{Div}_0 (1 + g_1) & \quad \text{Div}_0 (1 + g_1)^2 \\
0 & \quad 1 & \quad 2 & \quad \ldots \\
\text{Div}_0 (1 + g_1)^N & \quad = \text{Div}_0 (1 + g_1)^N (1 + g_2) \\
N & \quad \text{Div}_N (1 + g_2) & \quad N+1 & \quad \ldots
\end{align*}
\]
Case 3: Differential Growth

We can value this as the sum of:

- an $N$-year annuity growing at rate $g_1$

\[
P_A = \frac{C}{R - g_1} \left[ 1 - \frac{(1 + g_1)^T}{(1 + R)^T} \right]
\]

- plus the discounted value of a perpetuity growing at rate $g_2$ that starts in year $N+1$

\[
P_B = \frac{\left( \frac{\text{Div}_{N+1}}{R - g_2} \right)}{(1 + R)^N}
\]
Case 3: Differential Growth

Consolidating gives:

\[
P = \frac{C}{R - g_1} \left[ 1 - \frac{(1 + g_1)^T}{(1 + R)^T} \right] + \frac{\left( \frac{\text{Div}_{N+1}}{R - g_2} \right)}{(1 + R)^N}
\]

Or, we can “cash flow” it out.
A Differential Growth Example

A common stock just paid a dividend of $2. The dividend is expected to grow at 8% for 3 years, then it will grow at 4% in perpetuity.

What is the stock worth? The discount rate is 12%.
With the Formula

\[
P = \frac{\$2 \times (1.08)}{0.12 - 0.08} \left[ 1 - \frac{(1.08)^3}{(1.12)^3} \right] + \frac{\left( \frac{\$2(1.08)^3(1.04)}{0.12 - 0.04} \right)}{(1.12)^3}
\]

\[
P = \$54 \times [1 - 0.8966] + \frac{\$32.75}{(1.12)^3}
\]

\[
P = \$5.58 + \$23.31 \quad P = \$28.89
\]
**With Cash Flows**

$$P_0 = \frac{2.16}{1.12} + \frac{2.33}{(1.12)^2} + \frac{2.52 + 32.75}{(1.12)^3} = 28.89$$

$$P_3 = \frac{2.62}{.08} = 32.75$$

The constant growth phase beginning in year 4 can be valued as a growing perpetuity at time 3.
Estimates of Parameters

- The value of a firm depends upon its growth rate, $g$, and its discount rate, $R$.
  - Where does $g$ come from?
    
    \[ g = \text{Retention ratio} \times \text{Return on retained earnings} \]
  
  - Retention ratio is the fraction of net income which retained
Where does $R$ come from?

- The discount rate can be broken into two parts.
  - The dividend yield
  - The growth rate (in dividends)
- In practice, there is a great deal of estimation error involved in estimating $R$. 
Using the DGM to Find R

• Start with the DGM:

\[ P_0 = \frac{D_0 (1 + g)}{R - g} = \frac{D_1}{R - g} \]

Rearrange and solve for \( R \):

\[ R = \frac{D_0 (1 + g)}{P_0} + g = \frac{D_1}{P_0} + g \]
Growth Opportunities

- Growth opportunities are opportunities to invest in positive NPV projects.
- The value of a firm can be conceptualized as the sum of the value of a firm that pays out 100% of its earnings as dividends and the net present value of the growth opportunities.

\[ P = \frac{EPS}{R} + NPVGO \]
DGM vs. NPVGO

• We have two ways to value a stock:
  – The dividend discount model
  – The sum of its price as a “cash cow” plus the per share value of its growth opportunities
The NPVGO Model: Example

Consider a firm that has EPS of $5 at the end of the first year, a dividend-payout ratio of 30%, a discount rate of 16%, and a return on retained earnings of 20%.

- The dividend at year one will be $5 \times 0.30 = $1.50 per share.
- The retention ratio is 0.70 ( = 1 - 0.30), implying a growth rate in dividends of 14% = 0.70 \times 20%.

From the dividend growth model, the price of a share is:

\[
P_0 = \frac{\text{Div}_1}{R - g} = \frac{$1.50}{0.16 - 0.14} = $75
\]
The NPVGO Model: Example

First, we must calculate the value of the firm as a cash cow.

\[ P_0 = \frac{\text{EPS}}{R} = \frac{\$5}{.16} = \$31.25 \]

Second, we must calculate the value of the growth opportunities.

\[ P_0 = \left[ -3.50 + \frac{3.50 \times .20}{.16} \right] = \frac{\$875}{.16 - .14} = \$43.75 \]

Finally,

\[ P_0 = 31.25 + 43.75 = \$75 \]
Price-Earnings Ratio

- Many analysts frequently relate earnings per share to price.
- The price-earnings ratio is calculated as the current stock price divided by annual EPS.
  - *The Wall Street Journal* uses last 4 quarter’s earnings

\[
P/E \text{ ratio} = \frac{\text{Price per share}}{\text{EPS}}
\]
## Stock Market Reporting

<table>
<thead>
<tr>
<th>52 WEEKS</th>
<th>VOL</th>
<th>NET</th>
</tr>
</thead>
<tbody>
<tr>
<td>HI</td>
<td>LO</td>
<td>STOCK SYM</td>
</tr>
<tr>
<td>25.72</td>
<td>18.12</td>
<td>Gap Inc</td>
</tr>
</tbody>
</table>

- Gap has been as high as $25.72 in the last year.
- Gap has been as low as $18.12 in the last year.
- Gap pays a dividend of 18 cents/share.
- Given the current price, the dividend yield is .8%.
- Given the current price, the PE ratio is 18 times earnings.
- 3,996,100 shares traded hands in the last day’s trading.
- Gap ended trading at $21.35, which is unchanged from yesterday.