Returns

- **Dollar Returns**
  
  the sum of the cash received and the change in value of the asset, in dollars.

Percentage Returns

- the sum of the cash received and the change in value of the asset divided by the initial investment.
Dollar Return = Dividend + Change in Market Value

percentage return = \( \frac{\text{dollar return}}{\text{beginning market value}} \)

= \( \frac{\text{dividend} + \text{change in market value}}{\text{beginning market value}} \)

= dividend yield + capital gains yield
Returns: Example

• Suppose you bought 100 shares of Wal-Mart (WMT) one year ago today at $25. Over the last year, you received $20 in dividends (20 cents per share × 100 shares). At the end of the year, the stock sells for $30. How did you do?

• Quite well. You invested $25 × 100 = $2,500. At the end of the year, you have stock worth $3,000 and cash dividends of $20. Your dollar gain was $520 = $20 + ($3,000 – $2,500).

• Your percentage gain for the year is: $20.8\% = \frac{520}{2,500}$
Returns: Example

Dollar Return:

$520 gain

Percentage Return:

\[ 20.8\% = \frac{\$520}{\$2,500} \]
Holding Period Returns

• The holding period return is the return that an investor would get when holding an investment over a period of $n$ years, when the return during year $i$ is given as $r_i$:

$$\text{holding period return} = \left(1 + r_1\right) \times \left(1 + r_2\right) \times \cdots \times \left(1 + r_n\right) - 1$$
Holding Period Return: Example

• Suppose your investment provides the following returns over a four-year period:

<table>
<thead>
<tr>
<th>Year</th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10%</td>
</tr>
<tr>
<td>2</td>
<td>-5%</td>
</tr>
<tr>
<td>3</td>
<td>20%</td>
</tr>
<tr>
<td>4</td>
<td>15%</td>
</tr>
</tbody>
</table>

Your holding period return =

\[
(1 + r_1) \times (1 + r_2) \times (1 + r_3) \times (1 + r_4) - 1
\]

\[
= (1.10) \times (.95) \times (1.20) \times (1.15) - 1
\]

\[
= .4421 = 44.21\%
\]
Holding Period Returns

- A famous set of studies dealing with rates of returns on common stocks, bonds, and Treasury bills was conducted by Roger Ibbotson and Rex Sinquefield.

- They present year-by-year historical rates of return starting in 1926 for the following five important types of financial instruments in the United States:
  - Large-company Common Stocks
  - Small-company Common Stocks
  - Long-term Corporate Bonds
  - Long-term U.S. Government Bonds
  - U.S. Treasury Bills
Return Statistics

- The history of capital market returns can be summarized by describing the:
  - average return
    \[ \bar{R} = \frac{(R_1 + \cdots + R_T)}{T} \]

  the standard deviation of those returns

  \[ SD = \sqrt{VAR} = \sqrt{\frac{(R_1 - \bar{R})^2 + (R_2 - \bar{R})^2 + \cdots + (R_T - \bar{R})^2}{T - 1}} \]

  - the frequency distribution of the returns
## Risk & Return

### Historical Returns, 1926-2004

<table>
<thead>
<tr>
<th>Series</th>
<th>Average Annual Return</th>
<th>Standard Deviation</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large Company Stocks</td>
<td>12.3%</td>
<td>20.2%</td>
<td><img src="image" alt="Distribution" /></td>
</tr>
<tr>
<td>Small Company Stocks</td>
<td>17.4</td>
<td>32.9</td>
<td><img src="image" alt="Distribution" /></td>
</tr>
<tr>
<td>Long-Term Corporate Bonds</td>
<td>6.2</td>
<td>8.5</td>
<td><img src="image" alt="Distribution" /></td>
</tr>
<tr>
<td>Long-Term Government Bonds</td>
<td>5.8</td>
<td>9.2</td>
<td><img src="image" alt="Distribution" /></td>
</tr>
<tr>
<td>U.S. Treasury Bills</td>
<td>3.8</td>
<td>3.1</td>
<td><img src="image" alt="Distribution" /></td>
</tr>
<tr>
<td>Inflation</td>
<td>3.1</td>
<td>4.3</td>
<td><img src="image" alt="Distribution" /></td>
</tr>
</tbody>
</table>

Source: © *Stocks, Bonds, Bills, and Inflation 2006 Yearbook™*, Ibbotson Associates, Inc., Chicago (annually updates work by Roger G. Ibbotson and Rex A. Sinquefield). All rights reserved.
Stock Returns and Risk-Free Returns

- The *Risk Premium* is the added return (over and above the risk-free rate) resulting from bearing risk.
- One of the most significant observations of stock market data is the long-run excess of stock return over the risk-free return.
  - The average excess return from large company common stocks for the period 1926 through 2005 was: $8.5\% = 12.3\% - 3.8\%$
  - The average excess return from small company common stocks for the period 1926 through 2005 was: $13.6\% = 17.4\% - 3.8\%$
  - The average excess return from long-term corporate bonds for the period 1926 through 2005 was: $2.4\% = 6.2\% - 3.8\%$
Risk Premia

• Suppose that The Wall Street Journal announced that the current rate for one-year Treasury bills is 5%.

• What is the expected return on the market of small-company stocks?

• Recall that the average excess return on small company common stocks for the period 1926 through 2005 was 13.6%.

• Given a risk-free rate of 5%, we have an expected return on the market of small-company stocks of 18.6% = 13.6% + 5%
The Risk-Return Tradeoff

- T-Bills
- T-Bonds
- Large-Company Stocks
- Small-Company Stocks

Annual Return Average vs. Annual Return Standard Deviation

- T-Bills: Low risk, low return
- T-Bonds: Moderate risk, moderate return
- Large-Company Stocks: High risk, high return
- Small-Company Stocks: Very high risk, very high return

Prof. Ali Nejadmalayeri
Risk Statistics

• There is no universally agreed-upon definition of risk.

• The measures of risk that we discuss are variance and standard deviation.
  – The standard deviation is the standard statistical measure of the spread of a sample, and it will be the measure we use most of this time.
  – Its interpretation is facilitated by a discussion of the normal distribution.
Normal Distribution

- A large enough sample drawn from a normal distribution looks like a bell-shaped curve.

The probability that a yearly return will fall within 20.2 percent of the mean of 12.3 percent will be approximately 2/3.
Normal Distribution

• The 20.2% standard deviation we found for large stock returns from 1926 through 2005 can now be interpreted in the following way: if stock returns are approximately normally distributed, the probability that a yearly return will fall within 20.2 percent of the mean of 12.3% will be approximately 2/3.
### Example – Return and Variance

<table>
<thead>
<tr>
<th>Year</th>
<th>Actual Return</th>
<th>Average Return</th>
<th>Deviation from the Mean</th>
<th>Squared Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.15</td>
<td>.105</td>
<td>.045</td>
<td>.002025</td>
</tr>
<tr>
<td>2</td>
<td>.09</td>
<td>.105</td>
<td>-.015</td>
<td>.000225</td>
</tr>
<tr>
<td>3</td>
<td>.06</td>
<td>.105</td>
<td>-.045</td>
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</tr>
<tr>
<td>4</td>
<td>.12</td>
<td>.105</td>
<td>.015</td>
<td>.000225</td>
</tr>
<tr>
<td>Totals</td>
<td></td>
<td></td>
<td>.00</td>
<td>.0045</td>
</tr>
</tbody>
</table>

Variance = .0045 / (4-1) = .0015  
Standard Deviation = .03873
More on Average Returns

- Arithmetic average – return earned in an average period over multiple periods
- Geometric average – average compound return per period over multiple periods
- The geometric average will be less than the arithmetic average unless all the returns are equal.
- Which is better?
  - The arithmetic average is overly optimistic for long horizons.
  - The geometric average is overly pessimistic for short horizons.
Geometric Return: Example

- Recall our earlier example:

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</table>

Geometric average return =

\[(1 + r_g)^4 = (1 + r_1) \times (1 + r_2) \times (1 + r_3) \times (1 + r_4)\]

\[r_g = \frac{1}{4} \sqrt[4]{(1.10) \times (0.95) \times (1.20) \times (1.15)} - 1\]

\[= 0.095844 = 9.58\%\]

So, our investor made an average of 9.58% per year, realizing a holding period return of 44.21%.

\[1.4421 = (1.095844)^4\]
Geometric Return: Example

- Note that the geometric average is not the same as the arithmetic average:

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<tbody>
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</table>

Arithmetic average return = \( \frac{r_1 + r_2 + r_3 + r_4}{4} \)

\[= \frac{10\% - 5\% + 20\% + 15\%}{4} = 10\% \]
Forecasting Return

• To address the time relation in forecasting returns, use Blume’s formula:

\[ R(T) = \left( \frac{T - 1}{N - 1} \right) \times Geometric\ Average + \left( \frac{N - T}{N - 1} \right) \times Arithmetic\ Average \]

where, \( T \) is the forecast horizon and \( N \) is the number of years of historical data we are working with. \( T \) must be less than \( N \).