Volatility

FIN 6660 : Investments Seminar
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Sources of Volatility

- **Immediacy & Market Structure**
  - Stoll and Whaley (1990)

- **Dispersion of Belief**
  - Shalen (1993)

- **Transaction**
  - Jones, Kaul, and Lipson (1994)

- **Stochastic Volatility**
  - Heston (1993)
  - Lamoureux and Lastrapes (1993)
In NYSE, batch vs. continuous auctions
Role of specialist differs from open and other times of the day
Two scenarios:
- Lower volatility at the open if opening works like classical Walrasian auctions; prices bounce inside the bid-ask spread
- Higher volatility at the open if opening is a one-shot auction and specialist has more monopoly power at the open; price moves outside spread
Trading shock effect on Volatility:

- Imagine returns as

\[ r_t = e_t + (u_t - u_{t-1}) \]

- Then:

\[
\frac{\sigma^2(r_{O,t})}{\sigma^2(r_{C,t})} = \frac{\sigma^2(e_{O,t}) + \sigma^2(u_{O,t} - u_{O,t-1})}{\sigma^2(e_{C,t}) + \sigma^2(u_{C,t} - u_{C,t-1})} + 2 \frac{\text{Cov}(e_{O,t}, u_{O,t} - u_{O,t-1})}{\sigma^2(e_{C,t}) + \sigma^2(u_{C,t} - u_{C,t-1})} + 2 \frac{\text{Cov}(e_{C,t}, u_{C,t} - u_{C,t-1})}{\sigma^2(e_{C,t}) + \sigma^2(u_{C,t} - u_{C,t-1})}
\]
Bid-ask effect on Volatility:

- Imagine returns as

$$r_{O,t} = r_{D,t-1} + r_{N,t}$$
$$r_{C,t} = r_{D,t} + r_{N,t}$$

- Then:

$$\frac{\sigma^2(r_{O,t})}{\sigma^2(r_{C,t})} = \frac{\sigma^2(r_{D,t-1}) + \sigma^2(r_{N,t}) + 2 \text{Cov}(r_{D,t-1}, r_{N,t})}{\sigma^2(r_{D,t}) + \sigma^2(r_{N,t}) + 2 \text{Cov}(r_{D,t}, r_{N,t})}$$
Dispersion of beliefs lead to volatility
- Lead to excess volatility and excess volume
- Positive correlation between price changes and both contemporaneous and lagged volume
- Positive correlation between consecutive absolute price changes

Large literature on analysts’ forecast error and dispersion and stocks’ and options’ volatility
Occurrence of transition, that is the number of transaction, causes volume-volatility link

- Size of transaction has to independent value

Theoretical background:

- With information asymmetry, size of trade reflects quality of information or informedness of trades; insiders want to trade large on good information
- With information asymmetry, size of trade reflects strategic behavior of informed; insiders want to camouflage their by throwing on small trades first and then go large
Size and Occurrence of Trades:

- Imagine returns as

\[ r_{i,t} = \sum_{k=1}^{5} \alpha_{i,k} D_{k,t} + \sum_{j=1}^{12} \beta_j r_{i,t-j} + \epsilon_{i,t} \]

- Then:

\[ |\epsilon_{i,t}| = \alpha_i + \alpha_{im} M_t + \beta_i AV_{i,t} + \gamma_i N_{i,t} + \sum_{j=1}^{12} \rho_{i,j} |\epsilon_{i,t-j}| + \eta_{i,t} \]

If Monday = 1; else = 0

Average trade size = total shares traded by number of transactions

Number of transaction for security \(i\) on day \(t\)
How to model stochastic volatility empirically

- Options prices should not price volatility
- Time-series forecasts of volatility should be orthogonal to the implied volatility from an option pricing model

- Hull and White (1987) model of SV

\[
\begin{align*}
    dS &= \phi S \, dt + \sqrt{V} \, S \, dw \\
    dV &= \mu V \, dt + \xi V \, dz \\
    \text{Corr}(dw, dz) &\equiv \rho_{wz} = 0
\end{align*}
\]
GARCH(1,1)

- Discrete-time approximation of the diffusion processes, where:

\[ r_t = \bar{r} + \varepsilon_t \]

\[ \varepsilon_t | \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots \sim N(0, h_t) \]

\[ h_t = c + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} + \gamma \zeta_{t-1} \]

- If volatility is not priced, then \( \gamma = 0 \)

Black-Scholes implied volatility
Out-of-Sample forecasts

What explains realized volatility?

\[
z_t = \beta_0 + \beta_1 \zeta_t + \beta_2 G_t + \beta_3 H_t + \nu_t
\]

\[
z_t = \frac{1}{N} \sum_{j=1}^{N} \tilde{\epsilon}_{t+j}^2
\]

\[
G_t = \frac{1}{N} \sum_{j=1}^{N} h_{t+j}
\]

\[
H_t = \frac{1}{t} \sum_{i=1}^{t} \tilde{\epsilon}_i^2
\]
Asymmetric Volatility

- **Leverage**
  - Black (1976); Christie (1982)

- **Volatility Feedback**
  - Pindyck (1984); French, Schwert and Stambaugh (1987); Campbell and Hentschel (1992)

- **Integrated Approach**
  - Bekaert and Wu (2000)
  - Wu (2001)
Market Level Shocks:
\( P_{M,t}; r_{M,t}; \varepsilon_{M,t} \)

Firm Level Shocks:
\( P_{i,t}; r_{i,t}; \varepsilon_{i,t} \)

\( \sigma_{M,t+1} \)
\( E_t(r_{M,t+1}) \)

\( \sigma_{i,t+1} \)
\( E_t(r_{i,t+1}) \)

News

Leverage Effect
Persistence
Risk premium
Volatility feedback

\( \sigma^2_{i,t+1} \)

\( \sigma_{i M,t+1} \)

Leverage Effect
Persistence
Risk premium
Volatility feedback
Dividend growth and dividend volatility

Imagine:

\[ 1 = E_t [m_{t+1} R_{t+1}] \]

\[ m_{t+1} = \exp \left( -r^f - \frac{1}{2} \sigma_{m,t}^2 + \epsilon_{m,t+1} \right), \quad \epsilon_{m,t+1} | I_t \sim N(0, \sigma_{m,t}^2) \]

\[ R_{t+1} = (P_{t+1} + D_{t+1}) / P_t \]

\[ \begin{cases} g_{t+1} = \alpha_0 + \alpha_1 g_t + \epsilon_{d,t+1} \quad \epsilon_{d,t+1} | I_t \sim N(0, \sigma_{d,t}^2) \\ \sigma_{d,t+1}^2 = \beta_0 + \beta_1 \sigma_{d,t}^2 + \sigma_{d,t} \nu_{t+1} \quad \nu_{t+1} | I_t \sim N(0, \eta_{\nu}^2) \end{cases} \]
Empirical model of asymmetric volatility

We have:

\[
\begin{align*}
    r_{t+1} &= r^f + \lambda_1 \sigma_{d,t}^2 + \lambda_2 \varepsilon_{d,t+1} - \lambda_3 \sigma_{d,t} \nu_{t+1} \\
    g_{t+1} &= \alpha_0 + \alpha_1 g_t + \varepsilon_{d,t+1} \\
    \sigma_{d,t+1}^2 &= \beta_0 + \beta_1 \sigma_{d,t}^2 + \sigma_{d,t} \nu_{t+1} \\
    \varepsilon_{d,t+1} | I_t &\sim N\left(0, \sigma_{d,t}^2\right) \\
    \nu_{t+1} | I_t &\sim N\left(0, \eta_v^2\right)
\end{align*}
\]
Measuring Volatility

- **Non-Gaussian Measures:**
  - Standard Deviation
    - Related to return squared, chi-squared dist.
  - Log absolute
    - Highly non-Gaussian errors

- **Quasi-Gaussian Measures:**
  - Range-based
    - Alizadeh, Brandt, and Diebold (2002)
Distribution of Log Absolute Value of BM:

\[
\Pr[\ln|x_t| \in dy] = \frac{2e^y}{\sigma \sqrt{\tau}} \Phi\left(\frac{e^y}{\sigma \sqrt{\tau}}\right) dy
\]

Distribution of Log Range Value of BM:

\[
\Pr\left[\ln\left(\sup_{0<t \leq \tau} x_t - \inf_{0<t \leq \tau} x_t\right) \in dy\right] = 8 \sum_{k=1}^{\infty} (-1)^{k-1} \frac{k^2 e^y}{\sigma \sqrt{\tau}} \Phi\left(\frac{e^y}{\sigma \sqrt{\tau}}\right) dy
\]
Comparing moments:

<table>
<thead>
<tr>
<th></th>
<th>Log Abs. Ret.</th>
<th>Log Range</th>
<th>Normal Dist.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>(-0.64 + 0.50 \ln \tau + \ln \sigma)</td>
<td>(0.43 + 0.50 \ln \tau + \ln \sigma)</td>
<td></td>
</tr>
<tr>
<td>Std Dev.</td>
<td>1.11</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>Skewness</td>
<td>-1.53</td>
<td>0.17</td>
<td>0</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>6.93</td>
<td>2.80</td>
<td>3</td>
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