**Basic Model**

- **Price in efficient markets:**
  - \( v_t \) denote the (log) “fundamental value”
  - \( \mu_t = \mathbb{E}[v_t | H_t] \) the conditional value on public info \( H_t \)
  - \( p_t \) denote the (log) price of the risky asset at time \( t \)

Then, *in weakly efficient markets*:

\[
p_t = \mu_t
\]

\[
r_t = p_t - p_{t-1} = \varepsilon_t
\]

\[
\varepsilon_t = \mu_t - \mu_{t-1} = \mathbb{E}[v_t | H_t] - \mathbb{E}[v_{t-1} | H_{t-1}]
\]
Frictions

Price in presence of Bid-Ask:

- $s_t$ denote the bid-ask spread
- $x_t$ denote buy (+1) or sell (-1)

Then, in markets with this friction:

$$p_t = \mu_t + s_t; \quad s_t = s x_t$$

where $x_t = +1$, buy; $x_t = -1$, sell

$$r_t = \epsilon_t + s_t - s_{t-1} = \epsilon_t + s(x_t - x_{t-1})$$

$$\epsilon_t = E[v_t | H_t] - E[v_{t-1} | H_{t-1}]$$
Frictions (cont’d)

Price in presence of Bid-Ask:

- $s_t$ denotes the bid-ask spread
- $x_t$ denotes buy (+1) or sell (-1)

Then, price will be positively autocorrelated:

$$\text{Cov}(r_t, r_{t-1}) = -s^2$$

And so an estimate of true bid-ask is:

$$\hat{s} = 2\sqrt{\text{Cov}(r_t, r_{t-1})}$$
Private Information

- Price in presence of informed trader:
  - $s_t$ denotes the bid-ask spread
  - $x_t$ denotes signed order flow
  - $u_t$ denotes pure noise

Then, in markets with private information:

$$\varepsilon_t = \lambda x_t + u_t; \quad \lambda > 0$$

$$p_t - \mu_t = s + \lambda$$
Price Formation & Information

Inside the “Black Box” of Market

- From market makers standpoint
  - Provide Liquidity [see Demsetz (1968)]
  - Manage Inventory [see Garman (1976), Stoll (1978), Amihud & Mendelson (1980)]
  - Resolve Information Asymmetry [see Bagehot (1971)]
    - Glosten & Milgrom (1985)
    - Kyle (1981)
    - Holden and Subrahmanyam (1992)
    - Admati and Pfleiderer (1988)
    - Easley & O’Hara (1987)
Early Seminal Theories

- **Insider Trading & Information Revelation**
  - Kyle (Econ. 1985)
  - Holden & Subrahmanyan (JF 1992); multiple insiders
  - Admati and Pfleiderer (RFS 1988); strategic noise trading

- **Heterogeneous Beliefs & Prices**
  - Glosten & Milgrom (JFE 1985)

- **Learning Process of Market Makers & Price**

- **Trading, Volume & Volatility**
  - Foster & Viswanathan (RFS 1990; RFS 1993)
Seminal Empirics

- **Bid-Ask Spread**
  - Demsetz (QJE 1968)
  - Roll (JF 1984) and Amihud & Mendelson (JFE 1986)
  - Glosten & Harris (JFE 1988) and Huang & Stoll (RFS 1997)

- **Intraday Price Patterns**
  - French & Roll (JFE 1986) and many more

- **Dealers’ Problem from Transaction Data**
  - Ho and Macris (1984)
  - Hasbrouck (1988)
  - Madhavan and Smidt (1991)
Seminal Experimentals

- **Market Mechanism**
  - Schnitzlein (JF 1996)
  - Bloomfield (JF 1996)
  - Bloomfield & O’Hara (JF1999; RFS1999)
Determinants of Bid-Ask

Demsetz (1968)

- Market makers provide “predictable immediacy”; they are “providers of liquidity”, hence:

\[ s_i = \beta_0 + \beta_1 \ln(M_i) + \beta_2 (1/p_i) + \beta_3 \sigma_i + \beta_4 \ln(V_i) + \varepsilon_i, \]

- **Market Value**
- **Price**
- **Volatility**
- **Volume**
- **Average Bid-Ask**