Term Structure of Interest Rates: The Theories

1. Term Structure Facts

Looking at Figure 1, we observe two term structure facts.

- Fact 1: Interest rates for different maturities tend to move together over time.
- Fact 2: Yields on short-term bond more volatile than yields on long-term bonds.

![Figure 1: Term Structure of the Treasury Rates](image)

Figure 2, on the other hand, gives us the following term structure fact.

- Fact 3: Long-term yield tends to be higher than short term yields (i.e. yield curves *usually* are upward sloping).

![Figure 2: Treasury Yield Curve](image)
To explain these facts we shall make use of a three theories of term structure of interest rates. The *Expectations Theory*, which can explain Facts 1 and 2, cannot explain Fact 3 very well. The *Segmented Market Theory*, can explain Fact 3 but cannot explain Facts 1 and 2. The *Liquidity Premium Theory*, which combines both the expectations theory and the segmented market theory, can explain all the three facts.

2. The Expectations Theory

The key assumption behind this theory is that buyers of bonds do not prefer bonds of one maturity over another, so they will not hold any quantity of a bond if its expected return is less than that of another bond with a different maturity. Bonds that have this characteristic are said to be *perfect substitutes*. Note that what makes long term bonds different from the short term bonds are the inflation and interest rate risks. Therefore, this theory essentially assumes away inflation and interest rate risks.

To see what the assumption that bonds with different maturities are perfect substitutes leads to; consider the following two investment strategies:

1: Buy $1 of one-year bond (the short bond) and when it matures buy another one-year bond.

2: Buy $1 of two-year bond (the long bond) and hold it.

According to the expectations theory, they are perfect substitutes and their expected returns must be equal. To calculate these expected returns, first a few notations:

\[ i_t = \text{interest rate on 1-pd bond at } t, \]
\[ i_t^e = \text{expectation at } t \text{ about the interest rate on 1-pd bond at period (t+1),} \]
\[ i_{2t} = \text{interest rate on 2-pd bond at } (at t). \]

From Investment Strategy 1, the expected return,
\[ = (1 + i_t) + (1 + i_t) i_{t+1}^e - 1 \]
\[ = i_t + i_t^e + i_t (i_{t+1}^e) \]
\[ = i_t + i_{t+1}^e, \quad \text{because, } i_t (i_{t+1}^e) = 0. \]

From Investment Strategy 2, the expected return,
\[ = (1 + i_{2t}) + (1 + i_{2t}) i_{2t} - 1 \]
\[ = 2i_{2t} + (i_{2t})^2 \]
\[ = i_{2t}, \quad \text{because, } (i_{2t})^2 = 0. \]

By expectations theory,
\[ i_t + i_{t+1}^e = 2i_{2t}, \]
\[ \Rightarrow i_{2t} = \frac{i_t + i_{t+1}^e}{2}. \]

That is, interest rate on the long bond is the average of the interests on short bonds expected over the life of the long bond. More generally, for \( n \)-period bonds,
\[ i_{nt} = \frac{i_t + i_{t+1}^e + i_{t+2}^e + \ldots + i_{t+n-1}^e}{n}. \]
Consider a few numerical examples. Suppose, one-year rates over the next five years are 5%, 6%, 7%, 8% and 9%. Then, interest rate on the two-year long bond is
\[ \frac{5\% + 6\%}{2} = 5.5\% . \]
And, interest rate on five-year long bond is,
\[ \frac{5\% + 6\% + 7\% + 8\% + 9\%}{5} = 7\% . \]
Thus, interest rates for two to five year (long) bonds are, 5%, 5.5%, 6%, 6.5% and 7%, which, essentially, is the yield curve.

According to the expectations theory, long-term rates are all averages of expected future short-term rate. If \( i_t \) changes so will \( i_{nt} \) for \( n = 2, 3, 4, \ldots \), etc. Thus, interest rates of different maturities will move together (Fact 1). Also, an average smoothen out large volatilities. Therefore, if the current short-term rate changes (say, the expected short rate of just the next year), it will have very little impact on a long-term rate (say, the 10-year rate). Thus, short term rates are more volatile (Fact 2). Expectations theory cannot explain why long-term yields are normally higher than short-term yields, in other words, why the yield curve is usually upward sloping. If the short rates are low now, they are expected to go up in the future. In that case the yield curve will be upward sloping. On the other hand, if the short rates are high now, they are expected to go down and in that case the yield curve will be downward sloping. Now, at a given point in time, short-term yields are as likely to be high as they are to be low. Therefore, they are as likely to go up as they are to go down in the future. That means that the expectations theory predicts that the yield curves are as likely to be upward sloping as they are to be downward sloping. Thus, the expectations theory cannot explain why the yield curve is usually upward sloping.

3. **Segmented Markets Theory**

This theory assumes that markets for different-maturity bonds are completely segmented. The interest rate for each bond with a different maturity is then determined by the supply of and demand for the bond with no effects from the expected returns on other bonds with other maturities. In other words, longer bonds that have associated with them inflation and interest rate risks are completely different assets than the shorter bonds. Thus, the bonds of different maturities are not substitutes at all, so the expected returns from a bond of one maturity has no effect on the demand for a bond of another maturity.

Because bonds of shorter holding periods have lower inflation and interest rate risks, segmented market theory predicts that yield on longer bonds will generally be higher, which explains why the yield curve is usually upward sloping (Fact 3).

However, since markets for different-maturity bonds are completely segmented, there is no reason why the short and long yields should move together. And, because of the same reason, the segmented market theory also cannot explain why the short-term yields should be more volatile than the longer-term yields.
4. The Liquidity Premium Theory

Throughout our discussion of bonds, we emphasized that even the default-free bonds are risky because of uncertainty about inflation and future interest rates. The reason for the increase in inflation risk over time is clear-cut. The bondholders care about the purchasing power of the return – the real return – they receive from bonds, not just the nominal dollar value of the coupon payments. Uncertainty about inflation creates uncertainty about a bond’s real return, making the bond a risky investment. The further we look into the future, the greater the uncertainty about the level of inflation, which implies that a bond’s inflation risk increases with its time to maturity.

Interest-rate risk arises from a mismatch between investor’s investment horizon and a bond’s time to maturity. If a bondholder plans to sell a bond prior to maturity, changes in the interest rate generate capital gains or losses. The longer the term of the bond, the greater the price changes for a given change in interest rates and the larger the potential for capital losses. As in case of inflation, the risk increases with the term to maturity, so the compensation must increase as with it. The buyer of long-term bonds would require compensation for the risks they are taking buying long-term bonds.

The liquidity premium theory views bonds of different maturities as substitutes, but not perfect substitutes. Investors prefer short rather than long bonds because they are free of inflation and interest rate risks. Therefore, they must be paid positive liquidity (term) premium, \( l_m \), to hold long-term bonds. The yield, therefore, has two parts, one that is risk free (given by the formula according to the expectations theory) and another that is a premium for holding a longer-term bond. Thus, the yield on an \( n \)-period bond is,

\[
i_n = \frac{i_t + i_{t+1} + i_{t+2} + \ldots + i_{t+(n-1)}}{n} + l_m.
\]

As a numerical example, consider the following one-year interest rates over the next five years: 5%, 6%, 7%, 8% and 9%. Suppose investors’ preferences for holding short-term bonds gives us following liquidity premiums for one to five-year bonds, respectively: 0%, 0.25%, 0.5%, 0.75% and 1.0%. Then, interest rate on the two-year bond is,

\[
\frac{5\% + 6\%}{2} + 0.5\% = 5.75\%.
\]

And, interest rate on the five-year bond is,

\[
\frac{5\% + 6\% + 7\% + 8\% + 9\%}{5} + 1.0\% = 8\%.
\]

Interest rates on one to five-year bonds, thus, are: 5%, 5.75%, 6.5%, 7.25% and 8%. As you can see, liquidity premium theories produce yield curves that more steeply upward sloped.

Like the expectations theory, the liquidity premium theory predicts that interest rates of different maturities will move together because the long-term rates are essentially tied to the short-term rates. Long rates will also be less volatile because part of the long rate, which is just an average of the short rates, will smoothen out the volatility in the short rates. And finally, since the risk premium increases with time to maturity, the liquidity premium theory tells us that the yield curve will normally slope upwards, only rarely will it lie flat or slope downwards.
As an illustration, suppose that the short rates are expected to be the same for the next 30 years. As Figure 3 shows, the expectation theory will predict a flat yield curve, while the liquidity premium theory will predict an upward sloping yield curve.

![Figure 3: Yield Curves Predicted by Two Theories](image)

Figure 3: Yield Curves Predicted by Two Theories

Suppose that the short rates are expected to fall throughout the 30 years in the future. In that case the yield curve predicted by the expectation theory will be downward sloping but the yield curve predicted by the liquidity premium theory can still be upward sloping. In Figure 4, when short rates are expected decline, the yield curves will shift.
(black → red). Even when the yield curve from Expectations theory is negatively sloped, the yield curve from Liquidity Premium theory can still be positively sloped.

This handout has been prepared using Frederic Mishkin’s *Money, Banking, and Financial Markets* (8th edition), and Stephen Cecchetti’s *Money, Banking, and Financial Markets*.