Analyzing the Interest Rate

- Determination of: \( i = \text{nominal interest rate} \)
- Model of interest rate determination
  - DD-SS of Assets: Loanable Funds Framework
  - Liquidity Preference Framework
- The way we’ll go about each model
  - Setup of the model
  - Comparative Statics: \textit{ceteris paribus} condition
  - Implications of the model

Approach to Explain \( i \)

- Price of bond and \( i \) are negatively related
  - So, explanation of Price of bond → explain \( i \)
- The general approach:
  - Asset price → theory of interest rate determination
- Models of asset demand and supply
- Interest rates on different securities move together
  - we start with one type of security & one interest rate
  - DD-SS model for one year discount bond
  - Explain comovements of various interest rates (later)
- An alternative model: Liquidity preference model
Demand-Supply Model of Asset Pricing

- Demand for assets come from household (provider of funds)
- 2 decisions:
  - Whether to buy and hold asset ‘a’
  - Whether to buy another asset ‘b’ instead of ‘a’
- Supply of assets come from firms (need funds)
- Demand and supply determine asset price

Asset Demand

- Asset is a store of values
- Asset demand function, 
  \[
  \text{Asset demand} = f \left( \begin{array}{c}
  \text{Asset price,} \\
  \text{Expected return,} \\
  \text{Wealth,} \\
  \text{Risk,} \\
  \text{Liquidity}
  \end{array} \right) \text{ shift factors on (price,quantity) plane}
  \]
- Demand curve on (price, quantity) plane
  - Changes in price
    → movement along the curve (law of demand)
  - Changes in other determinants of demand
    → shift of the curve
Determinants of Asset Demand

- **Wealth**
  - Availability of funds

- **Expected Return**
  - Expected return =
    - (probability of state 1 happening) x (return in state 1) + (probability of state 2 happening) x (return in state 2)
  - ex:
    - Expected return last year: 0.5 x10 + 0.5 x5% = 7.5%
    - Expected return this year: 0.5 x15% + 0.5 x5% = 10%

- **Risk**
  - Compare two assets with same expected return, one with higher risk
    - asset ‘a’: 0.5 x15% + 0.5 x5% = 10%
    - asset ‘b’: 0.5 x10% + 0.5 x10% = 10% [risk free]
  - Hollywood and sports clichés

- **Liquidity**
  - Depth and breadth of the market
  - Lower transaction cost
  - Ex: a house is not a very liquid asset
Determinants of Asset Demand

### Table 1: Response of the Quantity of an Asset Demanded to Changes in Wealth, Expected Returns, Risk, and Liquidity

<table>
<thead>
<tr>
<th>Variable</th>
<th>Change in Variable</th>
<th>Change in Quantity Demanded</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wealth</td>
<td>↑</td>
<td>↑</td>
</tr>
<tr>
<td>Expected return relative to other assets</td>
<td>↑</td>
<td>↑</td>
</tr>
<tr>
<td>Risk relative to other assets</td>
<td>↑</td>
<td>↓</td>
</tr>
<tr>
<td>Liquidity relative to other assets</td>
<td>↑</td>
<td>↑</td>
</tr>
</tbody>
</table>

Note: Only increases in the variables are shown. The effect of decreases in the variables on the change in demand would be the opposite of those indicated in the rightmost column.

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Bond Price and Interest Rate

- One year discount bond,

\[ i = \frac{F - P}{P} \]

- \( i \) = interest rate = yield to maturity
- \( F \) = face value = $1000
- \( P \) = price of the bond

- So, if \( P=950 \), \( i=5.3\% \)
- \( P=750 \), \( i=33.0\% \)
Bond Price & Quantity Demanded: An Example

- Law of demand (an example)
  
  Point A: \( P = $950, B^d = $100 \text{ billion}, \ i = 5.3\% \)
  
  Point B: \( P = $900, B^d = $200 \text{ billion}, \ i = 11.1\% \)
  
  Point C: \( P = $850, B^d = $300 \text{ billion}, \ i = 17.6\% \)
  
  Point D: \( P = $800, B^d = $400 \text{ billion}, \ i = 25.0\% \)
  
  Point E: \( P = $750, B^d = $500 \text{ billion}, \ i = 33.0\% \)

- Movements: \{P↑, B^d↓, i↓\}, \{P↓, B^d↑, i↑\}
- Demand curve is \( B^d \) connecting A, B, C, D, E

Bond Demand on (price, quantity) plane

Bond Demand: \( B^d \)

- Demand curve is \( B^d \) connecting A, B, C, D, E
- \( i \) goes the opposite direction to \( P \)
- \( B^d \) downward sloping
- Positive relationship between \( B^d \) and \( i \): reward of lending is \( i \)
Bond Supply on (price, quantity) plane

**Bond Supply: \( B^s \)**
- Higher price of bond encourages more bonds to be issued
- \( B^s \) upward sloping
- Negative relationship between \( B^s \) and \( i \): cost of borrowing is \( i \)

### Bond Market Equilibrium

**Market Equilibrium**
- Occurs when \( B^d = B^s \), at \( P^* = $850 \), \( i^* = 17.6\% \)
- When \( P = $950 \), and \( i = 5.3\% \), \( B^d > B^s \)
  (excess supply): \( P \downarrow \) to \( P^* \), \( i \uparrow \) to \( i^* \)
- When \( P = $750 \), and \( i = 33.0\% \), \( B^d < B^s \)
  (excess demand): \( P \uparrow \) to \( P^* \), \( i \downarrow \) to \( i^* \)
Demand Shift: Wealth

- wealth $\uparrow \Rightarrow \begin{cases} \text{demand for bonds } \uparrow \text{ at the same price} \\ \text{or, supply of loanable funds } \uparrow \text{ at the same } i \\ \Rightarrow B^d \text{ curve shifts right} \\ \Rightarrow \downarrow i, \uparrow \text{ equilibrium quantity of bonds} \end{cases}$

- Factors that change wealth (and thereby $B^d$)
  - Business cycle expansion $\rightarrow \uparrow$ wealth
    - Recessions $\rightarrow \downarrow$ wealth
  - $\uparrow$ MPS $\rightarrow \uparrow$ wealth
    - $\downarrow$ MPS $\rightarrow \downarrow$ wealth

Demand Shift: Risk

- risk $\uparrow \Rightarrow \begin{cases} \text{demand for bonds } \downarrow \text{ at the same price} \\ \text{or, supply of loanable funds } \downarrow \text{ at the same } i \\ \Rightarrow B^d \text{ curve shifts left} \\ \Rightarrow \uparrow i, \downarrow \text{ equilibrium quantity of bonds} \end{cases}$

- Price of bond become volatile $\rightarrow \uparrow$ risk of bond
- Price of alternative asset (e.g. stock) becomes less volatile $\rightarrow \uparrow$ risk of bond
Demand Shift: Liquidity

liquidity $\uparrow$ $\Rightarrow$ \begin{align*}
&\text{demand for bonds } \uparrow \text{ at the same price} \\
&\text{or, supply of loanable funds } \uparrow \text{ at the same } i \\
\Rightarrow &\quad B^d \text{ curve shifts right} \\
\Rightarrow &\quad \downarrow i, \; \uparrow \text{equilibrium quantity of bonds}
\end{align*}

$\blacksquare$ More people trading in the bond market
$\rightarrow$ liquidity of bond $\uparrow$

$\blacksquare$ Rise of brokerage commission of stock trading
$\rightarrow$ compared to stock liquidity of bond $\uparrow$

Demand Shift: Expected Return ($R_e$)

Expected return $\uparrow$ $\Rightarrow$ \begin{align*}
&\text{demand for bonds } \uparrow \text{ at the same price} \\
&\text{or, supply of loanable funds } \uparrow \text{ at the same } i \\
\Rightarrow &\quad B^d \text{ curve shifts right} \\
\Rightarrow &\quad \downarrow i, \; \uparrow \text{equilibrium quantity of bonds}
\end{align*}

$\blacksquare$ What factors affect $R_e$

$\square$ $R_e$ on other assets
$\square$ Expected inflation ($\pi^e$)
$\square$ Expected future interest rate
Demand Shift: Expected Return
Relative to Other Assets

- Expected return from bonds remains constant but
  Expected return from other (alternative) assets ↑
  → expected return from bonds relative to other assets ↓
  → demand for bonds ↓
  → demand for bonds curve shift left
  → ↑ \( i \), ↓ equilibrium quantity of bonds

Example: Optimism in the stock market
  → ↑ expected return on stocks (an alternative to bonds)

Demand Shift: Expected Inflation

- Since bonds are not real goods and services
  how does expected inflation affect it?
  → through real assets (house, automobiles)

- Increase in \( \Pi^e \)
  → ↑ expected capital gain on real assets
  → \( R^e \) on real assets relative to bonds ↑
  → demand for bonds ↓
  → demand for bonds curve shift left
  → ↑ \( i \), ↓ equilibrium quantity of bonds
Demand Shift: Expected Return
Expected Future Interest Rate

- Time sequence

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>maturity (at T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>t</td>
<td>t+1</td>
<td></td>
</tr>
<tr>
<td>(i_t, P_t)</td>
<td>(i_{t+1}, P_{t+1})</td>
<td>(i_T, F_T)</td>
</tr>
</tbody>
</table>

- For a 1-year bond i and \( R^e \) are the same and future interest rate doesn’t matter current i
- When holding period=T, again, future interest doesn’t matter
- \( R^e \) may differ from i for other (long term) bonds as,

\[
R^e = \frac{C + P'_{t+1} - P_t}{P_t}
\]

→ ↑ expected future interest rate
→ ↓ expected future bond price → ↓ expected capital gain
→ \( R^e \) ↓

---

Demand Shift: Expected Return
Expected Future Interest Rate

- Expected future interest rate (in long term bonds) ↑
  → expected future price (of longer term) bonds ↓
  → expected return on bonds ↓
  → dd for bonds in the current period ↓
  → current i ↑
- Expected future interest rate (in long term bonds) ↓
  → expected future price (of longer term) bonds ↑
  → expected return on bonds ↑
  → dd for bonds in the current period ↑
  → current i ↓
### Shift in Supply of Bonds

- **Profitability of investment opportunities ↑**
  - → firms more willing to borrow
  - → supply of bonds (dd for loanable funds) ↑
  
  *Ex: Business cycle expansion → ↑ supply of bonds*

- **Expected inflation ↑**
  - → real interest rate falls
  - → real cost of borrowing falls
  - → supply of bonds (dd for loanable funds) ↑

- **US treasury issues bonds to finance govt. deficit**
  
  *Ex: Government deficit → ↑ supply of bonds*
Shifts in the Bond Supply Curve

- ↑ Profitability of Investment Opportunities
- ↑ Expected Inflation ($\pi^e$)
- ↑ Government Deficits

Changes in $\pi^e$: the Fisher Effect

If $\pi^e \uparrow$
- Relative $R^e \downarrow$, $B^d$ shifts in to left
- Real cost of borrowing ↓, $B^s \uparrow$, $B^s$ shifts out to right
- $P \downarrow$, $i \uparrow$

So, $(\pi^e, i)$ should move together
Fisher Effect Evidence in US

**Business Cycle Expansion**

- Wealth ↑, \( B^d ↑, B^f \) shifts out to right
- Profitable Investment opportunities ↑, \( B^f ↑, B^d \) shifts out to right
- If \( B^f \) shifts more than \( B^d \) then \( P \downarrow, i \uparrow \)

Do we see \( i \uparrow \) with expansion and vice versa?

- Interest Rate, \( i \)
  - \((i\) increases \( \downarrow \))
- Price of Bonds, \( P \)
  - \((P\) increases \( ↑ \))

Diagram showing the relationship between the price of bonds, interest rates, wealth, and investment opportunities during business cycle expansion.
Business Cycles & Interest Rates

Interest rates rose during business cycle expansions, fell during recessions

Converting to Loanable Funds Framework

Just rotating the demand curve
The Loanable Funds Framework

- Demand for bonds = Supply of loanable funds
- Supply of bonds = Demand for loanable funds
- Not restricted to 1-yr discount bonds

Liquidity Preference:
An Alternative Model

**Keynes’s Assumption:** Two Categories of Assets to store Wealth, \{Money, Bonds\}

i.e., total available wealth: \[ W = M^s + B^s \]

**Implication of the assumption:**

- Budget Constraint: \[ B^d + M^d = W \]
- Therefore: \[ B^d + M^d = M^s + B^s \]

Subtract \((M^d + B^s)\) both sides: \[ B^d - B^s = M^s - M^d \]

**Money Market Equilibrium:** \[ M^d = M^s \]

but, \[ M^d = M^s \Rightarrow B^d = B^s \]

i.e. [money market equilibrium] \[\Rightarrow\]

[bond market also in equilibrium]
Liquidity Preference Model
Derivation of Money Demand Curve

- Why do people hold money?
  - Transaction demand for money
  - Precautionary demand for money
  - Speculative demand for money
- Keynes assumption: return from money = 0
- Return on bonds is $i$. Thus, opportunity cost of holding money is $i$.
- As $i \uparrow$, relative $R_e$ on money ↓ (equivalently, opportunity cost of money ↑) ⇒ $M^d$ ↓
- Demand curve for money has a downward slope on {interest rate, quantity of money} plane

Liquidity Preference Model
Derivation of Money Supply Curve

- The central bank controls $M^s$
- It is a fixed amount (exogenous)
- $M^s$ curve is a vertical line
Money Market Equilibrium

Market Equilibrium
1. Occurs when \( M^d = M^s \), at \( i^* = 15\% \)
2. If \( i = 25\% \), \( M^s > M^d \) (excess supply): Price of bonds ↑, \( i \) ↓ to \( i^* = 15\% \)
3. If \( i = 5\% \), \( M^d > M^s \) (excess demand): Price of bonds ↓, \( i \) ↑ to \( i^* = 15\% \)

Rise in Income or the Price Level

1. Income (or P level) ↑, \( M^d ↑ \), \( M^d \) shifts out to right
2. \( M^d \) unchanged, \( i^* \) rises from \( i_1 \) to \( i_2 \)

Income ↑ → ↑ demand more money to buy more stuff
Price level ↑ → ↑ demand more money to buy the same stuff
A Note on Business Cycles

- Business cycle expansion
  - increase in income levels
  - increase in interest rates
  - clear prediction by the liquidity preference model

- In Loanable funds model the prediction is not clear

Rise in Money Supply

1. $M^s \uparrow$, $M^s$ shifts out to right
2. $M^d$ unchanged
3. $i^*$ falls from $i_1$ to $i_2$
Liquidity Preference vs Loanable Funds Model: Similarities

- **Loanable funds**: equating supply and demand for bonds
- **Liquidity preference**: equating supply and demand for money (which is equivalent to equating supply and demand for bonds)
- Most (but not all) predictions are similar
Liquidity Preference vs Loanable Funds Model: Differences

- **Liquidity preference**: has only two assets, hence ignores effects of expected returns from real assets (e.g. house) on interest rate.
- **Loanable funds**: easier to use when addressing the effects of expected inflation.
- **Liquidity preference**: easier to use when addressing effects of changes in income, the price level, and the supply of money.