Studying Different Interest Rates

In the last section we studied determination of one interest rate. But, the economy has many interest rates.

- Why are interest rates on different bonds different?
- How do different interest rates behave with respect to each other?

Risk Structure of Interest Rates, Term Structure of Interest Rates

- Risk structure of interest rate: Why do bonds with same term of maturity have different interest rates?
  - Default Risk
  - Liquidity
  - Tax rules

- Term structure of interest rate: What effects do terms of maturity have on interest rates
  - Theories of term structure of interest rates
  - Inflation risk and interest rate risk
Risk Structure of Long-Term Bonds in the United States

Observations to be Explained

- Corporate bonds have higher interest rates than the government bonds
- Corporate Baa bonds have higher interest rates than corporate Aaa bonds
- State and local government bonds have lower interest rates than US government (Treasury) bonds (*most of the time*)
Risk Structure: Default Risk

- Default Risk
- Risk premium: Default risk creates a ‘spread’ between the interest rates of the two bonds
- Using Loanable Funds model to analyze default risk
- Compare \(B^d, B^s\) of two assets
  1. Corporate bond
  2. ‘Default free’ US treasury bond

Default Risk in Loanable Funds Model

**Suppose.** Initially corporate bond does not have any default risk (perhaps government was backing the corporation at the initial period). So, both bonds have same interest rate initially.

Then, introduce default risk for the corporate bond
- \(\uparrow\) risk of corporate bond \(\rightarrow D^c\) shifts left
- \(\downarrow\) relative risk to treasury bond \(\rightarrow D^T\) shifts right
- on risk premium \((F - i^T)\)
Default Risk in Loanable Funds Model
Analysis of the Illustration

- **Corporate Bond Market**
  1. Risk of corporate bonds \( \uparrow \), \( D_c \) shifts left
  2. \( P_c \downarrow \), \( i_c \uparrow \)

- **Treasury Bond Market**
  3. Relative risk of Treasury bonds \( \downarrow \), \( D_T \) shifts right
  4. \( P_T \uparrow \), \( i_T \downarrow \)

- **Outcome**
  Risk premium, \( j_c - i_T \), rises
  Same reason why Baa bonds have higher \( i \) than Aaa bonds

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**Table 1: Bond Ratings by Moody's and Standard & Poor's**

<table>
<thead>
<tr>
<th>Rating</th>
<th>Standard &amp; Poor's</th>
<th>Description</th>
<th>Examples of Corporations with Bonds Outstanding in 2004</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A</td>
<td>Upper medium grade</td>
<td>Honeywell International Inc., General Dynamics, Boeing Company</td>
</tr>
<tr>
<td>Baa</td>
<td>BBB</td>
<td>Medium grade</td>
<td>ITT Industries, Northrop Grumman Corporation, Goodrich Corporation, FedEx Corporation</td>
</tr>
<tr>
<td>Ba</td>
<td>BB</td>
<td>Lower medium grade</td>
<td>Allied Waste Industries, IKON Office Solutions, Inc.</td>
</tr>
<tr>
<td>C</td>
<td>D</td>
<td>Lowest grade</td>
<td>US Airways, United Airlines, Citation Corp.</td>
</tr>
</tbody>
</table>
Risk Structure of Long-Term Bonds in the United States

Considerations of Liquidity

Treasury bond more liquid than corporate bond
- T-bonds: widely traded
- Corporate bonds: fewer bonds of any one corporation traded

- **Corporate Bond Market**
  1. Less liquid corporate bonds $D_c \downarrow$, $D_c$ shifts left
  2. $P_c \downarrow$, $i_c \uparrow$

- **Treasury Bond Market**
  1. Relatively more liquid Treasury bonds, $D_T \uparrow$, $D_T$ shifts right
  2. $P_T \uparrow$, $i_T \downarrow$

- **Outcome** ‘Liquidity’ premium, $\hat{i} - i_T$, rises
  - Convention: called ‘Risk premium’
Liquidity Consideration in Loanable Funds Model

Suppose, Initially corporate bond does not have any default risk and liquidity is also the same as treasury bonds. So, both bonds have same interest rate initially. Then, corporate bonds become less liquid → interest spread ($i_c - i_T$)

Income Tax Considerations: Tax Advantages of Municipal Bonds

- Interest payment on municipal bonds are exempt from federal taxes
- Explaining using Loanable Funds Model
  1. Tax exemption raises relative $R^e$ on municipal bonds, $D^m \uparrow$, $D^m$ shifts right
  2. Relative $R^e$ on Treasury bonds $\downarrow$, $D^T \downarrow$, $D^T$ shifts left
  3. Outcome: $i^m < i^T$
- Before WWII income tax rates were extremely low ⇒ the tax-exempt status didn’t matter (explains the historical trend)
Tax Advantages of Municipal Bonds

What makes bonds risky?

- **Default risk**
  - The issuer may not make the promised payment
- **Inflation risk**
  - Inflation may reduce the real return on bonds
- **Interest rate risk**
  - Interest rates may turn out to be higher than expected (arises from the mismatch between investors horizon and bond’s maturity date)
Term Structure

- Refers to the relationship among bonds of same characteristics but different maturity

- **Yield curve:** A representation (plot) of the term structure
  - **x-axis:** time to maturity
  - **y-axis:** yield to maturity

Term Structure of Treasury Interest Rates

- The interest rates tend to move together over time
- Yields on short-term bond more volatile then long-term bonds
The yield curve usually have upward slope. The theory must explain that. However, occasionally the yield curve does have a negative slope (inverted yield curve). Therefore, the theory must establish mechanism that determines the slope of the yield curve.

Sufia J, 7/9/2006
Yield Curves on the Newspaper

- Yield curve is usually upward sloping

![Treasury Yield Curve](source: Reuters)

Term Structure Facts

- **Fact 1**: Interest rates for different maturities tend to move together over time
- **Fact 2**: Yields on short-term bond more volatile than yields on long-term bonds
- **Fact 3**: Long-term yield tends to be higher than short term yields (i.e. yield curve usually positively sloped).
The yield curve usually have upward slope. The theory must explain that. However, occasionally the yield curve does have a negative slope (inverted yield curve). Therefore, the theory must establish mechanism that determines the slope of the yield curve.
Theories of Term Structure

Three Theories of Term Structure
- Expectations Theory
  - Explains Fact 1 and Fact 2, NOT Fact 3
- Segmented Markets Theory
  - Explains Fact 3, NOT Fact 1 and Fact 2
- Liquidity Premium Theory
  - Combines features of both Expectations Theory and Segmented Markets Theory to get Liquidity Premium
  - Preferred Habitat Theory
  - Explain all 3 facts

Expectations Theory
- **Key Assumption**
  Bonds of different maturities are perfect substitutes (i.e. there is no inflation or interest rate risks)

- **Implication**
  $R_o$ on bonds of different maturities equal for Investment strategies for 2-period horizon:
  - **Investment strategy 1:**
    Buy $1 of one-year bond and when it matures buy another one-year bond
  - **Investment strategy 2:**
    Buy $1 of two-year bond and hold it
Expectations Theory

Notation

\( i_t \) = interest rate on 1-pd bond at \( t \)

\( i^e_{t+1} \) = expectation at \( t \) about the interest rate on 1-pd bond at period \((t+1)\)

\( i_{2t} \) = interest rate on 2-pd bond (at \( t \))

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Expectations Theory

Investment Strategy 1:

\[
R^e = (1 + i_t) + (1 + i_t) i^e_{t+1} - 1
\]

\[
= i_t + i^e_{t+1} + i_t(i^e_{t+1})
\]

\[
\approx i_t + i^e_{t+1} \quad \text{[because, } i_t(i^e_{t+1}) \approx 0]\]

Investment Strategy 2:

\[
R^e = (1 + i_{2t}) + (1 + i_{2t})(i_{2t}) - 1
\]

\[
= 2i_{2t} + (i_{2t})^2
\]

\[
\approx 2i_{2t} \quad \text{[because, } (i_{2t})^2 \approx 0]\]
Expectations Theory

\( R^e \) from both the investment strategies being equal:
\[
\Rightarrow \quad i_t + i^e_{t+1} = 2i_{2t}
\]
\[
\Rightarrow \quad i_{2t} = \frac{i_t + i^e_{t+1}}{2}
\]

More generally, for the interest rate of \( i_{nt} \) on a
an \( n \) - period bond,
\[
i_{nt} = \frac{t_i + i^e_{t+1} + i^e_{t+2} + \ldots + i^e_{t+(n-1)}}{n}
\]

**Interest rate on long bond**
= average short rates expected over life of long bond

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Expectations Theory: Numerical Example

One-year interest rate over the next five years:
5%, 6%, 7%, 8% and 9%:

Interest rate on two-year bond:
\( (5\% + 6\%)/2 = 5.5\% \)

Interest rate on five-year bond:
\( (5\% + 6\% + 7\% + 8\% + 9\%)/5 = 7\% \)

Interest rate for one to five year bonds (yield curve):
5%, 5.5%, 6%, 6.5% and 7%.

**Note:** 1-yr rates are increasing \( \rightarrow \) rising yield curve
Expectations Theory and Term Structure Facts

Fact 1: \[ i_{nt} = \frac{i_t + i_{t+1} + i_{t+2} + \ldots + i_{t+n-1}}{n} \]

Long-term rates are all averages of expected future short-term rate. If \( i_t \) changes so will \( i_{nt} \) for \( n=2,3,4, \ldots \) etc. Thus, interest rates of different maturities will move together.

Fact 2: Average smoothens out large volatilities.

ex: average of \{5, 9, 6, 8, 7\} = 7

If current short-term rate changes (say, 3-month rate), it will have very little impact on a long-term rate (say, 10-year rate). Thus, short term rates are more volatile.

CANNOT EXPLAIN Fact 3.

- When short rates are low, they are expected to rise to normal level, and yield curve will have steep upward slope
  Ex: Short: 5, 10, 15, 20, Long: 5, 7.5, 10, 12.5
- When short rates are high, they will be expected to fall in future, and yield curve will have downward slope
  Ex: Short: 20, 15, 10, 5, Long: 20, 17.5, 15, 12.5
- Short rates as likely to fall in future as they are to rise. There is no guarantee that yield curve will not usually slope upward
Segmented Markets Theory

- **Key Assumption**
  Bonds of different maturities are not substitutes at all

- **Implication**
  - Markets are completely segmented: interest rate at each maturity are determined separately
  - Inflation risk and Interest rate risk ⇒ People typically prefer short holding periods and thus have higher demand for short-term bonds

Segmented Market Theory and Term Structure Facts

- **Cannot explain Fact 1 or Fact 2**
  Because short and long-term rates are determined completely independently
  - No reason why Short and long-term rates should move together
  - Cannot explain why short-rates should be more volatile than long rates

- **Explains Fact 3: yield curve typically upward sloping**
  People prefer short holding periods
  ⇒ short-term bonds have higher demand
  ⇒ short-term bonds have higher price (and lower interest rates) than long-term bonds
Liquidity Premium Theory

Modifies Expectations Theory with features of Segmented Markets Theory

- **Key Assumption**
  Bonds of different maturities are substitutes, but are not perfect substitutes

- **Implication**
  - people prefer short rather than long bonds
  - people must be paid positive term premium, $l_{nt}$ (called *Liquidity premium*) to hold long-term bonds

Thus,

$$i_{nt} = \frac{i_1 + i_{n+1} + i_{n+2} + \ldots + i_{n+(n-1)}}{n} + l_{nt}$$

Numerical Example

- One-year interest rate over the next five years:
  5%, 6%, 7%, 8% and 9%

- Households prefer holding short-term bonds,
  liquidity premiums for one to five-year bonds:
  0%, 0.25%, 0.5%, 0.75% and 1.0%.

- Interest rate on the two-year bond:
  \( \frac{5\% + 6\%}{2} + 0.25\% = 5.75\% \)

- Interest rate on the five-year bond:
  \( \frac{5\% + 6\% + 7\% + 8\% + 9\%}{5} + 1.0\% = 8\% \)

- Interest rates on one to five-year bonds:
  5%, 5.75%, 6.5%, 7.25% and 8%

- Comparing with those for the expectations theory, liquidity premium (preferred habitat) theories produce yield curves more steeply upward sloped
Liquidity Premium (Preferred Habitat) and Expectations Theories

Suppose short rates are expected to remain the same.

Liquidity Premium vs Expectations Theories

- Suppose short rates are expected to decline, then yield curves will shift (black → red).
- Even when yield curve from Expectations theory is negatively sloped, yield curve from Liquidity Premium theory can still be positively sloped.
Liquidity Premium Theory
Explains all 3 term Structure Facts

- Fact 1: Explanation similar to Expectations theory
- Fact 2: Explanation same as Expectations theory
- Fact 3: Explanation similar to Segmented markets theory
  - households have a preference for short-term bond-holding (inflation and interest rate risks)
  - Because of the risk premium, even if interest rates are not expected to rise, long-term rate will typically exceed the short-term rate
- Note: This theory can also explain the occasional ‘inverted yield curves’. It happens when short rates are expected to go down a lot offsetting the risk premium