The \textit{IS-LM} Model

- Underlying model due to \textit{John Maynard Keynes}
- Model representation due to \textit{John Hicks}
- Used to make predictions about
  - Interest rates
  - Aggregate spending (= aggregate output)
- Important assumption: price is fixed
  - The ‘Fixed Price’ model
  - Essentially deals with short run determination of output and interest rate

The Sequence

- Output determination in the goods market
- Goods market equilibrium condition: IS
- Money market equilibrium condition: LM
- Objective: analyzing effects of policies on interest rate and aggregate income
  - \textit{Fiscal policy}: control of government spending and taxes
  - \textit{Monetary policy}: control of interest rate and money supply
Keynes: The Context

- How government policy could be used to increase employment in situation similar to the GD
- Emphasis of the demand side
- Not restricted to analyzing GD-type catastrophes
  - Macroeconomic model to understand movements in aggregate output in short-run
  - Macroeconomic stabilization policies

Aggregate Demand

- Total quantity demanded of an economy’s output is the sum of four types of spending:
  \[ Y^{ad} = C + I + G + NX \]
- Equilibrium occurs when: \( Y = Y^{ad} \)
  \( \Rightarrow \) producers can sell their outputs and have no reason to change their production
- Emphasis on Effective demand
  - if \( Y^{ad} \) is not enough (\( Y > Y^{ad} \)) the economy is producing more than there is demand for
    \( \Rightarrow \) output below the “full employment” level
  - To understand output fluctuations, need to understand the components of aggregate demand
Consumption Function (C)

- Disposable income: \( Y_D = Y - T \)
  \( Y \) = aggregate income = aggregate output
  \( T \) = taxes
- The consumption function: \( C = a + (mpc \times Y_D) \)
  \( a \) = autonomous consumption expenditure
  = consumption spending when \( Y_D \) is equal to 0
  \( mpc \) = marginal propensity to consume = \( \frac{\Delta C}{\Delta Y_D} \)
- Why is autonomous consumption positive?
  Examples: students, wealth, unemployment benefits.
- Ex: \( mpc = 0.8 \) ⇒ if \( Y_D \) increases by $1, then 80 cents out of that will be spent, and 20 cents saved
- Keynes assumed: \( mpc \) is a constant

**TABLE 1** Consumption Function: Schedule of Consumer Expenditure C When \( mpc = 0.5 \) and \( a = 200 \) ($ billions)

<table>
<thead>
<tr>
<th>Point in Figure 1</th>
<th>Disposable income ( Y_D ) (1)</th>
<th>Change in Disposable Income ( \Delta Y_D ) (2)</th>
<th>Change in Consumer Expenditure ( \Delta C ) ((0.5 \times \Delta Y_D)) (3)</th>
<th>Consumer Expenditure ( C ) (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>0</td>
<td>—</td>
<td>—</td>
<td>200 ((= a))</td>
</tr>
<tr>
<td>F</td>
<td>400</td>
<td>400</td>
<td>200</td>
<td>600</td>
</tr>
<tr>
<td>G</td>
<td>800</td>
<td>400</td>
<td>200</td>
<td>800</td>
</tr>
<tr>
<td>H</td>
<td>1,200</td>
<td>400</td>
<td>200</td>
<td>800</td>
</tr>
</tbody>
</table>

**FIGURE 1** Consumption Function
Planned Investment Spending (I)

- Economist’s view of Investment
  - Adding something NEW (ex: new machine)
  - Buying of assets such as stocks are NOT investment

- Fixed Investment
  - Spending by firms on equipments (ex: machines) and structures (ex: office buildings)
  - Spending on residential housing

- Inventory Investment
  - Spending by firms on additional holdings of intermediate and finished goods = (holdings at the beginning of period – holding at the end of the period)

Planned versus Unplanned

- Fixed investment always planned
- Inventory investment can be
  - Planned \( (I_p) \): part of \( I \), hence part of \( Y_{ad} \)
  - Unplanned \( (I_u) \): not part of \( I \), hence not part of \( Y_{ad} \)

- Only Planned items constitute \( I \), the component of \( Y_{ad} \).
  \[
  I = (\text{fixed investment}) + (\text{planned inventory investment})
  \]

- Planned investment spending depends on
  - Interest rates
  - Business’s expectations about the future
If $T=0$, $Y=Y_D$
$C = a + mpc.Y$

Suppose, $I = 300$
Note that $I$ does not depend on Disposable Income

$C = 200 + 0.5Y_D$

$Y^{ad} = C + I + G + NX$
Simple model:
let, $G = 0$, $NX = 0$
$\Rightarrow Y^{ad} = C + I$

**FIGURE 2** Keynesian Cross Diagram
Output Response
to Changes in $Y^{ad}$ components

- Simple model: $Y^{ad} = C + I$, \[ \therefore G = 0, \quad NX = 0 \]
- How change in $C$ and $I$ (and later on, other components) change aggregate output?
- What kind of changes to be expected?
  - Changes in investment
  - Changes in autonomous consumption
  - Why not changes in consumption other than autonomous consumption?
  - NOTE: later on we’ll also discuss changes in other components of $Y^{ad}$. For simple model only $C$ and $I$.
- A rise in $a$ or $I$ shifts the $Y^{ad}$ upwards
- Magnitude of output response: multiplier

Output Response: Change in $I$

![Graph showing output response to change in investment](image)

**Figure 3** Response of Aggregate Output to a Change in Planned Investment
Expenditure Multiplier

- Components of aggregate demand: \( Y^{ad} = C + I \)
  - In equilibrium: output, \( Y = Y^{ad} = C + I \)
  - \( I \) does not depend on \( Y \) but, \( C = f(Y) \)
- When \( \Delta I \Rightarrow \) initial effect is, \( \Delta Y = \Delta I \)
- Chain of events follow after the initial effect
  - As there is an increase in \( Y \), we have \( C \) increased
  - As \( C \) increases, \( Y \) also increases, again, and so on
- The final effect on \( Y \) (=\( \Delta Y \)) higher than the initial \( \Delta I \)

Expenditure Multiplier: Example

- Suppose, mpc=0.5, for everyone
- Consider an exogenous $1000 ↑ in Sara’s income (say, due to some investment spending by some firm)
- Initial increase in income is $1000
- Chain of events spending $1000
- Final increase in income = $2000

<table>
<thead>
<tr>
<th>Chain of Events</th>
<th>Spending (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sara buys from Hasan</td>
<td>500</td>
</tr>
<tr>
<td>Hasan buys from Geraldo</td>
<td>250</td>
</tr>
<tr>
<td>Geraldo buys from Tyrone</td>
<td>125</td>
</tr>
<tr>
<td>Tyrone buys from Inga</td>
<td>63</td>
</tr>
<tr>
<td>Inga buys from Paolo</td>
<td>31</td>
</tr>
<tr>
<td>Paolo buys from Nigel</td>
<td>16</td>
</tr>
<tr>
<td>Nigel buys from Ravi</td>
<td>8</td>
</tr>
<tr>
<td>Ravi buys from Avi</td>
<td>4</td>
</tr>
<tr>
<td>Avi buys from Ahtunowhiho</td>
<td>2</td>
</tr>
<tr>
<td>Ahtunowhiho buys from Takeshi</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td>1000</td>
</tr>
</tbody>
</table>
Expenditure Multiplier: Derivation

- $Y = Y_{old} = C + I$
  
  but, $C = a + (mpc \times Y)$
  
  so, $Y = a + (mpc \times Y) + I$
  
  $\Rightarrow Y - (mpc \times Y) = a + I$

- LHS $= Y - (mpc \times Y) = Y(1 - mpc)$

- $\therefore$ we have,
  
  $Y(1 - mpc) = a + I$
  
  $\Rightarrow Y = \frac{a + I}{1 - mpc}$

Expenditure Multiplier: Derivation

- old $Y = \frac{a + I}{1 - mpc}$, (i.e., before $\Delta I$ took place)

  $\Delta I \Rightarrow$ new $Y = \frac{a + (I + \Delta I)}{1 - mpc}$

  $\Delta Y = \text{new } Y - \text{old } Y = \frac{a + (I + \Delta I)}{1 - mpc} - \frac{a + I}{1 - mpc} = \frac{\Delta I}{1 - mpc}$

- $mpc > 0$ and $mpc < 1$

  $\therefore 1 - mpc < 1$, $\Rightarrow \frac{1}{1 - mpc} > 1$

  $\therefore \frac{\Delta I}{1 - mpc} = \left(\frac{1}{1 - mpc}\right) \times \Delta I > \Delta I$

- Example: $mpc = 0.5$, $1 - mpc = 0.5$, $\frac{1}{1 - mpc} = 2$

  $\Delta I = 100$, $\frac{100}{0.5} = 200 > 100$
Expenditure Multiplier

Income is,

\[ Y = a + I + (mpc \times Y) \]

\[ \Leftrightarrow Y = \frac{1}{1 - mpc} \times (a + I) \]

After an increase in investment spending,

\[ \Delta Y = \frac{\Delta I}{1 - mpc} \]

Note that,

\[ \Delta Y \text{ (output increase)} = \frac{1}{1 - mpc} \times \Delta I \text{ (expenditure increase)} \]

What if there is an increase in \( a \) instead. Say, \( \Delta a \)?

Expenditure Multiplier

- \[ Y = \frac{1}{1 - mpc} \times (a + I) \]

Could the same \( \Delta Y \) take place if, instead of \( \Delta I \), we have \( \Delta a \) of the same magnitude?

- Aggregate demand: \[ Y^{ad} = \frac{a + I}{\text{autonomous}} + (mpc \times Y) \]

- Any change in the autonomous component will have the same kind of effect where,

\[ \Delta Y \text{ (output increase)} = \frac{1}{1 - mpc} \times \Delta A \text{ (increase in autonomous spending)} \]
Full Model

\[ Y = Y^{ad} = a + \left( \frac{mpc \times Y}{c} \right) + I + G + NX \]

\[ Y = a + I + G + NX + \left( \frac{mpc \times Y}{\text{autonomous}} \right) \]

Alternative expression,

\[ Y = \frac{1}{1 - mpc} \times (a + I + G + NX) \]

\[ Y = \frac{1}{1 - mpc} \times A \]

Multiplier effect,

\[ \Delta Y = \frac{1}{1 - mpc} \times \Delta A \]

Multiplier,

\[ \frac{1}{1 - mpc} > 1 \]

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Expenditure Multiplier

Output Contraction

\[ \Delta Y = \frac{1}{1 - mpc} \times \Delta A \]

When autonomous spending increases by \( \Delta A \), output increase by \( \left( \frac{1}{1 - mpc} \right) \times \Delta A \)

What happens when autonomous spending DECREASES by \( \Delta A \)?

Output DECREASES by \( \left( \frac{1}{1 - mpc} \right) \times \Delta A \).

\[ \Rightarrow \text{A larger than } \Delta A \text{ decline in output.} \]
Keynes’s Explanation for GD

Keynes’s Prescription to GD

- Decline in autonomous spending
- Cannot rely on autonomous C or I to increase
- Increase G, under government’s control
- Fiscal Policy
Government: The Full Treatment

Consumption function: \( C = a + (mpc \times Y_D) \)

No taxes \( \Rightarrow \ Y_D = Y \)

With taxes, \( Y_D = Y - T \)

Consumption function with taxes,

\[
C = a + (mpc \times (Y - T)) = a - (mpc \times T) + (mpc \times Y)
\]

\[
Y_{ad} = \frac{1}{1 - mpc} \left[ a - (mpc \times T) + I + G \right]
\]

- ↑ Taxes \( \Rightarrow \) \( C \downarrow \)
- A $1 ↑ in taxes \( \Rightarrow \) \( C \downarrow \) by the amount of \( (mpc \times T) \)
  [Ex: if \( mpc=0.5 \), then $1 ↑ in taxes \( \Rightarrow \) \( C \downarrow \) by 50 cents]
- \( T \downarrow \) is an ↑ autonomous (like \( G \uparrow \)) but effect dampened by \( mpc \)

Government Spending vs Taxes

[FiguRe 5  Response of Aggregate Output to Government Spending and Taxes]
Explaining the Diagram

\[ Y^{ad} = C + I + G + NX \]

\[ C = a + (mpc \times (Y - T)) = a - (mpc \times T) + (mpc \times Y) \]

Consider the straight line on \((Y^{ad}, Y)\) plane,

\[ Y^{ad} = \left[ a - (mpc \times T) + I + G \right] + mpc \times Y \]

Example: \(mpc = 0.5, \uparrow G = 400, \uparrow T = 400\)
- \(\uparrow G \rightarrow \) intercept of \(Y^{ad}\) increases by 400
  \(\rightarrow Y^{ad}\) shifts up by 400
- \(\uparrow T \rightarrow \) intercept of \(Y^{ad}\) decreases by \((0.5 \times 400) = 200\)
  \(\rightarrow Y^{ad}\) shifts down by 200
- There is a net increase in \(Y\), in the equilibrium

The Interest Rate

- So far, we have talked about income determination, and what has been missing is, interest rate \((i)\)
- The connection between \(i\) and \(Y\) is investment \((I)\)
- We need to figure out: \(i \longleftrightarrow (I) \longleftrightarrow Y\)
- First we'll discuss investment schedule: \(i \longleftrightarrow (I)\)
- Then we'll use the Keynesian cross diagram
- Combining these two will give us: \(i \longleftrightarrow Y\)
The Investment Schedule

- For firms with no surplus funds, interest rate \((i)\) is the cost of borrowing
- For firms with surplus funds, they can put their funds in two things
  - Planned investment spending (which will yield a return)
  - Buy bonds (which will also yield a return)
    - If \((i)\) is high they are more likely to buy bonds
- Thus, in either case we have that,
  \[ I = I(i). \]
  In words: \(i\) and \(I\) are negatively related.
The Keynesian Cross Diagram
Changes in $Y$ Due to Changes in $I$

**FIGURE 7** Deriving the IS Curve (continued)
The IS Curve & the LM curve

- The IS curve gives us the combination of \((i, Y^*)\)
- The IS curve tells us what the associated level of \(i\) for each \(Y^*\).
  In other words: for each goods market equilibrium what is the level of interest rate associated with that?
- So, what we have is \((i, Y^*)\)
  Once we have \((i^*, Y)\),
  Combining these two we can have \((i^*, Y^*)\).
- The LM curve will give us \((i^*, Y)\). This comes from the money market equilibrium.

Money Market:
Liquidity Preference Model

- Keynesian money demand function,
  \[
  \frac{M}{P} = f(i, Y)
  \]
- Interest rate \((i)\) is the opportunity cost of holding money
- As income \((Y)\) increases, money demand increases
- Supply of money is a fixed exogenously given quantity
Derivation of the LM Curve

The IS Curve & the LM curve

- The LM curve gives us the combination of \((i^*, Y)\)
- The LM curve tells us what the associated level of \(Y\) for each \(i^*\).
  In other words: for each money market equilibrium what is the level of aggregate output associated with that?
- Now that we have the IS curve with \((i, Y^*)\), and the LM curve with \((i^*, Y)\), crossing them will give us the economy-wide \((i^*, Y^*)\).
ISLM Diagram: Simultaneous Determination of Output and the Interest Rate